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Σ^- atomic states and the nucleon distribution in Pb

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Abstract. The analyses of (K^-,π) and (π^-,K^+) reactions indicate that the nuclear potential of the Σ-hyperon is repulsive inside the nucleus, in agreement with the prediction of model F of the Nijmegen baryon-baryon interaction. This is consistent with the recent calculation of the strong-interaction shifts and widths of the observed levels of Σ^- atoms, including the precise data on the Σ^-Pb atom. In this paper, the sensitivity of this calculation to the neutron and proton density distributions is used to determine these densities in ^{208}Pb.

PACS. 13.75.Ev Hyperon-nucleon interactions – 36.10.Gv Mesonic atoms and molecules, hyperonic atoms and molecules

1 Introduction

The analyses [1–3] of the Brookhaven (K^-,π) experiments [4] and of the KEK (π^-,K^+) experiments [5] clearly lead to the conclusion that the nuclear potential of the Σ-hyperon is repulsive inside the nucleus, in agreement with the prediction of model F [6] of the Nijmegen baryon-baryon interaction[1].

As we have shown in [8] (hereafter referred to as I), this conclusion is consistent with properties of Σ^- atoms, *i.e.*, strong-interaction shifts ϵ and widths Γ of the lowest observed levels of Σ^- atoms, including the precise data on the Σ^-Pb atom [9]. In calculating ϵ and Γ in I we applied a definite form of the proton and neutron density distributions ρ_p and ρ_n in the nuclear core of the Σ^- atoms considered. The resulting values of ϵ and Γ are sensitive to the form of ρ_p and ρ_n.

In the present paper, we exploit this sensitivity to determine ρ_p and ρ_n in the ^{208}Pb nucleus. We choose this nucleus because of the relatively high accuracy of the Σ^- data of ref. [9]. Furthermore, the oversimplified nucleon densities in ^{208}Pb, applied in I, were not well founded.

We apply the following procedure: we start with Hartree-Fock (HF) densities ρ_p and ρ_n which we apply in calculating ϵ and Γ for the two lowest states observed in Σ^-Pb. We do it for three models of the Nijmegen ΣN interaction: models D [10] and F [6], and the soft-core model SC [11]. Next, we modify ρ_p and ρ_n to a final form which assures that the best agreement between the calculated and observed values of ϵ and Γ is achieved with model F. Namely, we know from the analysis of the (K^-,π) and (π,K^+) experiments that among the Nijmegen models only model F is consistent with these experiments.

Our computational procedure is outlined in sect. 2, and our results are presented and discussed in sect. 3.

2 The computational procedure

We calculate ϵ and Γ in the way described in detail in I. We solve the Schrödinger equation for the wave function Ψ of Σ^- in the atom:

$$[-(\hbar^2/2\mu)\Delta + V_{\rm C}(r) + V(r) + iW(r)]\Psi = \mathcal{E}\Psi, \qquad (1)$$

where μ is the Σ^--nucleus reduced mass and $V_{\rm C}$ is the Coulomb interaction between Σ^- and the nucleus. Because of the $\Sigma\Lambda$ conversion process $\Sigma^- p \to \Lambda n$, the strong-interaction potential $V + iW$ is complex and the eigenvalue $\mathcal{E}$ is also complex, $\mathcal{E} = E_{\rm C} - \epsilon - i\Gamma/2$, where $E_{\rm C}$ is the pure Coulomb energy.

To calculate the real and absorptive strong-interaction potentials V and W, we apply the local-density approximation in which the Σ^- atom is treated at each point as Σ^- moving in nuclear matter with the local nuclear density of the Σ^- atom:

$$V(r) = V^{\rm NM}(\rho_n(r),\rho_p(r)),$$
$$W(r) = W^{\rm NM}(\rho_n(r),\rho_p(r)), \qquad (2)$$

where $V^{\rm NM}(\rho_n,\rho_p)$ and $W^{\rm NM}(\rho_n,\rho_p)$ are the real and imaginary parts of the single-particle (s.p.) potential of

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[1] Let us mention also that model F applied to the Λ + nuclear-matter system, solves the so-called Λ overbinding problem [7].

Table 1. Energy shifts ϵ, $\epsilon^{\rm u}$ and widths Γ, $\Gamma^{\rm u}$ calculated with the indicated models of the ΣN interaction and the nucleon densities, respectively for the lower and upper level of the Σ^-Pb atom and the corresponding values of χ^2 for the 3 experimental Pb data. All energies are in eV.

	$V_{\Sigma N}$	ϵ	Γ	$\epsilon^{\rm u}$	$\Gamma^{\rm u}$	χ^2
A) HF$\longrightarrow \rho_n, \rho_p$ [a]	D	448.1	506.7	11.5	8.9	7.9
	F	24.7	236.6	2.4	6.0	65.2
	SC	250.1	367.7	7.7	7.5	19.6
B) $V_\tau = 0$	D	762.4	743.6	19.2	10,6	45.4
	F	349.9	300.2	13.6	7.0	13.4
	SC	235.5	380.5	6.8	7.6	21.0
C) $\rho_p \longrightarrow \bar{\rho}_p = \frac{Z}{N}\rho_n$	D	688.2	891.6	20.1	19.4	31.9
	F	266.6	495.0	12.7	15.3	8.2
	SC	268.6	626.6	8.8	16.6	9.1
D) $\bar{\rho_p} \longrightarrow \tilde{\rho}_p = \bar{\rho}_p + \delta\bar{\rho}_p$	D	658.2	770.6	19.9	18.3	22.7
	F	272.7	467.8	13.2	15.1	7.6
	SC	270.4	582.4	8.8	16.2	8.4
Experiment [b]		422 ± 56	428 ± 158		17 ± 3	

([a]) Taken from ref. [17].
([b]) Taken from ref. [9].

Σ^- in nuclear matter with neutron and proton densities ρ_n and ρ_p.

An expansion in terms of the neutron excess parameter $\alpha(r) = (\rho_n(r) - \rho_p(r))/\rho(r)$ $[\rho = \rho_n + \rho_p]$ leads to the splitting of V into the isoscalar potential V_0 and the isovector or Lane potential V_τ:

$$V(r) = V_0(r) + \frac{1}{2}\alpha(r)V_\tau(r),$$
$$V_0(r) = V_0^{\rm NM}(\rho(r)), \quad V_\tau(\rho(r)) = V_\tau^{\rm NM}(\rho(r)), \qquad (3)$$

where $V_0^{\rm NM}$ and $V_\tau^{\rm NM}$ are the isoscalar and Lane potentials in nuclear matter with total density ρ (and with $N = Z$). For $V_0^{\rm NM}$ and $V_\tau^{\rm NM}$ we use expressions in terms of the effective ΣN interaction derived in [12]. In the case of the Nijmegen baryon-baryon interaction models considered in this paper, we apply the effective YNG interaction obtained in [13] within the low-order Brueckner theory.

For the absorptive potential, we apply the semiclassical expression[2] $W^{\rm NM} \simeq -\frac{\hbar}{2}\rho_p\langle v\sigma\rangle$, where $\langle\ \rangle$ denotes averaging in the Fermi sea, v is the relative $\Sigma^- p$ velocity, and σ is the total cross-section for the $\Sigma^- p \to \Lambda n$ conversion process, for which we use the parametrization [14]: $(v/c)\sigma = (1 + 13v/c)^{-1} 5.1$ fm^2, see footnote [3].

3 Results and discussion

In the Σ^- ^{208}Pb atom there are three data points measured in [9]: the energy shift ϵ and the width Γ of the lower level with the principal and orbital quantum numbers $n = 9$, $l = 8$, and the width $\Gamma^{\rm u}$ of the upper level with $n = 10$, $l = 9$. They are shown in the bottom line in table 1. We want to find nucleon densities in ^{208}Pb, $\rho_p(r)$ and $\rho_n(r)$ which lead to the best agreement between the calculated and measured values of ϵ, Γ and $\Gamma^{\rm u}$. We do it in four steps, A)–D).

A) We start with HF densities calculated by Skalski [17] with the Skyrme interaction SkM*. They are shown in fig. 1 as the solid curves denoted as ρ_n and ρ_p. Our results for the strong-interaction shifts and widths of the lower and upper level in the Σ^-Pb atom, obtained with models D, F and SC of the Nijmegen interaction, are shown in table 1, together with the values of χ^2 calculated for the three experimental data points[4]. We see that the value of ϵ calculated with model F is much smaller than the experimental value. These results favor model D which —as discussed before— is incompatible with the results of the Brookhaven (K^-, π) and the KEK (π^-, K^+) experiments. We interpret it as an indication that the HF nucleon densities require modifications.

B) To get a hint on how to modify the HF densities, let us consider the effect of the Lane potential V_τ. As discussed in I, V_τ is repulsive and thus its presence diminishes the resulting value of ϵ. Consequently, if we disregard V_τ, the resulting value of ϵ should increase and improve the agreement of model F results with experiment. This is indeed so as seen in table 1. At the same time, the resulting value of ϵ for model D becomes too great compared to the experimental result. The situation with the SC model is slightly different because V_τ for this model is relatively weak, and at very low density it becomes even attractive. Anyhow, we see that in the absence of V_τ, model F leads to the best agreement with the experimental data.

[2] The expression actually used contains exclusion principle corrections and the nucleon effective mass.

[3] Our final conclusions would remain unchanged if we used for σ the parametrization of Oset *et al.* [15], discussed in [16].

[4] We do not consider the new soft-core model [18], because —as shown in [12]— it leads to an attractive V_τ inside the nucleus which is incompatible with the Brookhaven (K^-, π) experiments [4].

C) However, a relatively strong Lane potential is implied by the Nijmegen baryon-baryon interaction models (especially model F) and it cannot be disregarded as it is essential in interpreting the Brookhaven $(K^-, \pi^\pm)$ experiments. Instead, we may diminish its contribution $\frac{1}{2}\alpha(r)V_\tau(r)$ to $V(r)$ by diminishing the neutron excess parameter $\alpha(r)$ in the region of r essential in calculating ϵ and Γ. As discussed in I, this region for the lower state is around $r \sim 9$ fm, and here our HF densities lead to a substantial neutron excess, much bigger than the overall value $\bar{\alpha} = (N-Z)/A$. The simplest way to reduce $\alpha(r)$ to the value of $\bar{\alpha}$ is to replace the original HF proton density $\rho_p(r)$ by $\bar{\rho}_p(r) = \frac{Z}{N}\rho_n(r)$. And indeed —as is seen in table 1— with this form of the proton density distribution, model F leads to the best agreement with experiment[5].

D) Our proton density, $\bar{\rho}_p(r)$, leads to the root-mean-square (r.m.s.) radius of proton distribution $R_p[\bar{\rho}_p] = \langle r^2[\bar{\rho}_p]\rangle^{1/2} = 5.62$ fm, which is too big compared to the semi-empirical r.m.s. radius of (point) proton distribution $R_p^{\rm se} = 5.44$ fm (this value of $R_p^{\rm se}$ leads —after taking into account the r.m.s. proton charge radius of 0.8 fm— to the r.m.s. radius of the charge distribution $R_{\rm ch} = 5.50$ fm which is consistent with the r.m.s. charge radii of ^{208}Pb tabulated in [19] and in table XII in [20]). To reduce R_p to the semi-empirical value of 5.44 fm, without increasing the neutron excess in the peripheral region important in the Σ^-Pb problem, we shift some of the protons from the interior region close to the nuclear surface towards the nuclear center. We do it by adding $\delta\bar{\rho}_p$ to $\bar{\rho}_p$ obtaining our final proton density $\tilde{\rho}_p(r) = \bar{\rho}_p(r) + \delta\bar{\rho}_p(r)$. For $\delta\bar{\rho}_p$ we assume the form

$$\delta\bar{\rho}_p(r) = x_1 e^{-(r/w_1)^2} - x_2 e^{-((r-R_2)/w_2)^2}, \qquad (4)$$

where x_2 is determined by the condition: $4\pi\int {\rm d}r r^2 \delta\tilde{\rho}_p(r) = 0$. For the remaining parameters we assume the values $x_1 = 0.75(\rho_n(0) - \bar{\rho}_p(0))$, $R_2 = 6.66$ fm, $w_1 = 3.66$ fm, and $w_2 = 1.0$ fm. With these parameters, the r.m.s. radius of the proton distribution becomes equal to the semi-empirical radius, $R_p[\tilde{\rho_p}] = 5.44$ fm.

Our final neutron and proton density distributions $\rho_n(r)$ and $\tilde{\rho}_p(r)$ are shown in fig. 1. Results obtained with these densities for strong-interaction energy shifts and widths of the two lowest observed levels in Σ^-Pb, are presented in table 1. They clearly favor model F with the value of χ^2 per degree of freedom, $\chi^2/3 = 2.5$. One obtains very similar results if one applies the HF SkX densities calculated by Brown [21]. These densities and the corresponding proton density $\tilde{\rho}_p$ are presented in fig. 1 as dotted lines.

For the r.m.s. radius of the neutron distribution, we get $R_n[\rho_n] = \langle r^2[\rho_n]\rangle^{1/2} = 5.62$ fm, and for the neutron skin $S = R_n[\rho_n] - R_p[\tilde{\rho}_p] = 0.18$ fm. These results agree with the results obtained from the analyses of nucleon elastic scattering [22], of neutron equation of state [23], and of pionic atoms [24].

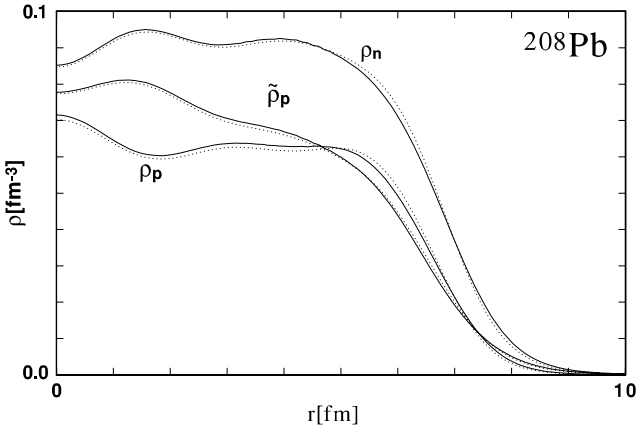

Fig. 1. The HF densities ρ_n, ρ_p of ref. [17] (solid curves) and ref. [21] (dotted curves), and the corresponding modified proton densities $\tilde{\rho}_p$.

Although our proton density $\tilde{\rho}_p(r)$ leads to the semi-empirical value of 5.44 fm of the r.m.s. radius, it differs from the proton densities implied by the charge density distributions tabulated in [19] and [20]. In the nuclear center, our $\tilde{\rho}_p$ is bigger than what is implied by the tabulated charge density distributions. This difference, however, appears not so important if one considers the more relevant quantity $r^2\rho(r)$. In the nuclear periphery relevant for the observed levels in the Σ^-Pb atom, the neutron excess $(\rho_n(r) - \tilde{\rho}_p(r))/(\rho_n(r) + \tilde{\rho}_p(r))$ is close to $\bar{\alpha} = (N-Z)/A$, whereas the neutron excess obtained with the HF densities approaches the value one. This is reflected in the values of the higher moments of our proton distribution, $\langle r^4\rangle^{1/4}$ and $\langle r^6\rangle^{1/6}$, which are greater than the HF values and the values implied by table IX in [20].

We conclude with the following comments:

– To reach our conclusions concerning the nucleon density distribution in ^{208}Pb, we have applied approximations, discussed in I, especially the local-density approximation.

– We rely on the experimental values of the energy shifts and widths of the two lowest levels in Σ^-Pb determined in [9]. It should be stressed that to determine the energy shifts, one has to know very accurately the energy levels of the hypothetical Σ^-Pb atom with a switched-off strong interaction, which is a nontrivial theoretical problem.

– The HF densities, which we applied, were obtained with Skyrme effective two-body interactions assumed to depend on a single nucleon density, $\rho = \rho_n + \rho_p$. However, in an asymmetric nuclear matter with an appreciable neutron excess, the dependence of the effective interaction on two densities, ρ_n and ρ_p, may be important [25], especially for the magnitude of the Lane s.p. nuclear potential [26], which plays a role in estimating the radius of the proton distribution [27].

[5] Replacing the original HF neutron density $\rho_n(r)$ by $\bar{\rho}_n(r) = \frac{N}{Z}\rho_p(r)$ would also reduce $\alpha(r)$ to the value of $\bar{\alpha}$, but at the same time the overall size of the density, including its tail, would be reduced too much. The agreement with experiment of model F would be only slightly improved, and model D would remain to be favored.

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